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Backus and Wyllie Averages for Seismic Attenuation

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Abstract—Backus and Wyllie equations are used to obtain average seismic velocities at zero and infinite frequencies, respectively. Here, these equations are generalized to obtain averages of the seismic quality factor (inversely proportional to attenuation). The results indicate that the Wyllie velocity is higher than the corresponding Backus quantity, as expected, since the ray velocity is a high-frequency limit. On the other hand, the Wyllie quality factor is higher than the Backus one, following the velocity trend, i.e., the higher the velocity (the stiffer the medium), the higher the attenuation. Since the quality factor can be related to properties such as porosity, permeability, and fluid viscosity, these averages can be useful for evaluating reservoir properties.

Key words: Wyllie equation, Backus averaging, seismic attenuation.

1. Introduction

The amplitude of seismic waves plays a key role in determining the properties of rocks. Amplitude variations highly depend on frequency since formations are generally heterogeneous and anelastic, especially those saturated with liquids and gas. Depending on the wavelength, the behaviour of signal differs (Pride et al. 2004; Mainardi 2010). A typical sedimentary rock environment consists of sequences of thin beds, where a thin bed can be defined as a stratigraphic unit that has a thickness less than the predominant wavelength. Seismic velocity and attenuation are affected by the layer thicknesses and intrinsic properties of the single layers. Carcione (1992) first described loss in fine layering, obtaining exact expressions numerically and analytically. Further progress has been achieved by Zhu et al. (2007), who developed approximate solutions by assuming that the velocity and attenuation contrasts, as well as the interval velocity- and attenuation-anisotropy parameters, are small by absolute value. At high frequencies, the medium is heterogeneous and the wavefield can be represented by rays. In this case, the so-called Wyllie time average equation holds (Wyllie et al. 1956). At low frequencies, the medium is effectively homogeneous and Backus averaging can be used to obtain the seismic properties (Backus 1962). These equations are well known regarding seismic velocity [e.g., Carcione et al. 1991; Stovas and Ursin (2007)], but the corresponding averages for attenuation have not been obtained to our knowledge.

The purpose of this note is to obtain averages for the seismic quality factor, Q, (inversely proportional to attenuation), at a given frequency or at the low- and high-frequency limits. We verify the Backus average using full-waveform numerical simulations. The modeling is based on the Zener mechanical model and the differential equations are solved in the space–time domain using a direct method based on the Fourier pseudospectral method (Carcione 1992, 2014).

The quality factor can be related to properties such as porosity, permeability, and fluid viscosity by means of the mesoscopic-loss mechanism (e.g., Carcione 2014). In fact, this mechanism seems to be the most important at seismic frequencies. For instance, for mesoscopic patches of gas in a water-saturated sandstone, diffusion of pore fluid in and out between different patches dissipates energy through conversion of energy to the diffusive slow mode (e.g.,

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Carcione 2014), and this conversion is highly affected by permeability. The increasing awareness in the scientific community that energy loss is providing useful micro-structural information implies that the expressions developed in this work will be important for seismic interpretation, e.g., when the average Q of a set of layers is required to be related to the properties of the single layers to obtain information about its lithology, porosity, permeability, or fluid type.

2. Backus Averaging

Let us assume n isotropic layers of thickness h_i, density ρ_i, and P-wave velocities c_i, and let $H = \sum_i h_i$ and $p_i = h_i/H$ be the total thickness of the formation and proportion of each single layer, respectively. Backus (1962) obtained the effective elastic constants of a finely layered medium in the long-wavelength case. The stiffness constant normal to the layering is as follows:

$$c_{33} = \left(\sum_i \frac{p_i}{\rho_i c_i^2} \right)^{-1}. \tag{1}$$

The equivalent medium is transversely isotropic. Let us now assume that the layers are anelastic based on the Zener model, each with a maximum quality factor Q_i at the frequency f_0 and that c_i is the unrelaxed (high-frequency) velocity. The complex velocity as a function of the frequency $f = \omega/(2\pi)$ is as follows:

$$v_i(f) = c_i \sqrt{\frac{\mathrm{i}f/f_0 + 1/a_i}{\mathrm{i}f/f_0 + a_i}}, \quad a_i = Q_i^{-1} + \sqrt{1 + Q_i^{-2}} \tag{2}$$

(e.g., Carcione 2014). The relaxed (low-frequency) velocity is c_i/a_i. By high frequency, we intend here a wavelength much larger than the thickness of the layers, i.e., the long-wavelength approximation still holds.

The complex stiffness is obtained by replacing Eq. (2) into Eq. (1):

$$p_{33} = \left(\sum_i \frac{p_i}{\rho_i v_i^2} \right)^{-1}. \tag{3}$$

The Backus average complex velocity is as follows:

$$v_{\mathrm{B}} = \frac{1}{\sqrt{\sum_i p_i \rho_i}} \left(\sum_i \frac{p_i}{\rho_i v_i^2} \right)^{-1/2}, \tag{4}$$

where we have assumed an arithmetic average for the density. The Backus phase velocity is $1/\mathrm{Re}(1/v_{\mathrm{B}})$ (Carcione 2014) (in the following "Re" and "Im" denote real and imaginary parts, respectively), that is

$$c_{\mathrm{B}} = \left[\mathrm{Re} \left(\sqrt{\sum_i p_i \rho_i \sum_i \frac{p_i}{\rho_i v_i^2}} \right) \right]^{-1}. \tag{5}$$

The relaxed and unrelaxed values are as follows:

$$c_{\mathrm{Br}} = \left(\sum_i p_i \rho_i \sum_i \frac{p_i a_i^2}{\rho_i c_i^2} \right)^{-1/2} \tag{6}$$

and

$$c_{\mathrm{Bu}} = \left(\sum_i p_i \rho_i \sum_i \frac{p_i}{\rho_i c_i^2} \right)^{-1/2}, \tag{7}$$

respectively.

For uniform density, Eq. (5) reads

$$c_{\mathrm{B}} = \left[\mathrm{Re} \left(\sqrt{\sum_i \frac{p_i}{v_i^2}} \right) \right]^{-1}. \tag{8}$$

The average-quality factor is as follows:

$$Q_{\mathrm{B}} = \frac{\mathrm{Re}(v_{\mathrm{B}}^2)}{\mathrm{Im}(v_{\mathrm{B}}^2)} = -\frac{\mathrm{Re}(v_{\mathrm{B}}^{-2})}{\mathrm{Im}(v_{\mathrm{B}}^{-2})} = -\frac{\sum_i \frac{p_i}{\rho_i} \mathrm{Re}(v_i^{-2})}{\sum_i \frac{p_i}{\rho_i} \mathrm{Im}(v_i^{-2})} \tag{9}$$

(Carcione 2014). The quality factor of each single layer at the frequency f_0 is as follows:

$$Q_i = -\frac{\mathrm{Re}(v_i^{-2})}{\mathrm{Im}(v_i^{-2})}. \tag{10}$$

3. Wyllie Equations

In the case that the signal wavelength is very short (infinite frequency), the travel time through the layers is as follows:

$$t = \frac{H}{c_W} = \sum_i \frac{h_i}{c_i}, \tag{11}$$

where

$$c_W = \left(\sum_i \frac{p_i}{c_i}\right)^{-1} \tag{12}$$

is the Wyllie average velocity.

Next, we propose a similar equation to obtain an average-quality factor of the stack of layers. A plane wave in a lossy medium attenuates as follows:

$$\Pi_i \exp(-\alpha_i h_i) \approx \Pi_i \exp\left(-\frac{\omega}{2c_i Q_i} h_i\right), \tag{13}$$

(e.g., Carcione 2014), where α_i is the attenuation factor of each layer. The approximation of the attenuation factor in Eq. (13) holds for a low-loss medium. An exact expression for any level of loss can be found in Carcione (2004, eq. 2.122). We may re-write (13) as follows:

$$\exp\left(-\frac{\omega}{2}\sum_i \frac{h_i}{c_i Q_i}\right), \tag{14}$$

or

$$A = \exp\left(-\frac{\omega H}{2 c_W Q_W}\right), \tag{15}$$

where

$$Q_W = \left(\frac{c_W}{H}\sum_i \frac{h_i}{c_i Q_i}\right)^{-1} = \left(c_W \sum_i \frac{p_i}{c_i Q_i}\right)^{-1} \tag{16}$$

is the average quality factor. Note that if we define the traveltime of each layer as $t_i = h_i/c_i$, the average or equivalent quality factor is as follows:

$$Q_W = \sum_i t_i / \sum_i \frac{t_i}{Q_i}, \tag{17}$$

i.e., the weighted average of the single Q factors where the weights are the transit times.

4. Full-Waveform Modeling Method

The full-waveform synthetic seismograms are computed with a modeling code based on the viscoacoustic stress–strain relation corresponding to a single relaxation mechanism, represented by the Zener mechanical model. The equations are given in Sect. 2.10.4 of Carcione (2014). The 1D particle velocity–stress formulation describing propagation along the x-direction is as follows:

$$\begin{aligned} \dot{v} &= \frac{1}{\rho}\partial_x \sigma, \\ \dot{\sigma} &= \rho c^2(\partial_x v_x + e) + s, \\ \dot{e} + \frac{e}{\tau_\sigma} &+ \left(\frac{2}{\tau_0 Q}\right)\partial_x v_x = 0, \end{aligned} \tag{18}$$

where v is particle velocity, σ is stress, s is the source (explosion), e is a memory variable, τ_σ denotes a relaxation time, and a dot above a variable indicates time differentiation. The relaxation time is as follows:

$$\tau_\sigma = \tau_0\left(\sqrt{Q^{-2}+1} - Q^{-1}\right), \tag{19}$$

where Q is the minimum quality factor and $\tau_0 = 1/2\pi f_0$. If f_p is the central frequency of the source wavelet, we assume that the relaxation peak is located at $\omega_0 = 1/\tau_0 = 2\pi f_p$. The velocity c in these equations corresponds to the unrelaxed or high-frequency limit velocity. The preceding equations hold for each single layer and for the equivalent medium.

The numerical algorithm is based on the Fourier pseudospectral method for computing the spatial derivatives and a fourth-order Runge–Kutta technique for calculating the wavefield recursively in time (e.g., Carcione 2014).

5. Results

Let us consider a periodic system of two layers with proportions p_1 and p_2, velocities c_1 = 2 km/s and c_2 = 2.5 km/s, densities ρ_1 = 2.1 g/cm^3 and ρ_2 = 2.3 g/cm^3, and quality factors Q_1 = 10 and Q_2, the values of which are given below. We assume f_0 = 50 Hz (relaxation peak) and a frequency $f = f_0$. Backus averaging assumes that the layer thicknesses are much smaller than the wavelength c_2/f_0 = 50 m, while the Wyllie wavelength is zero. Figure 1 shows the phase velocities as a function of proportion p_1 ($p_2 = 1 - p_1$) for Q_2 = 25. As can be seen, the Wyllie velocity is an upper limit. On the other hand. Figure 2

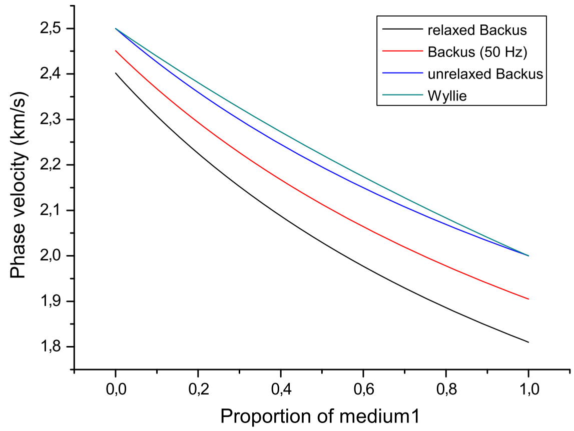

Figure 1
Backus and Wyllie phase velocities as a function of p_1

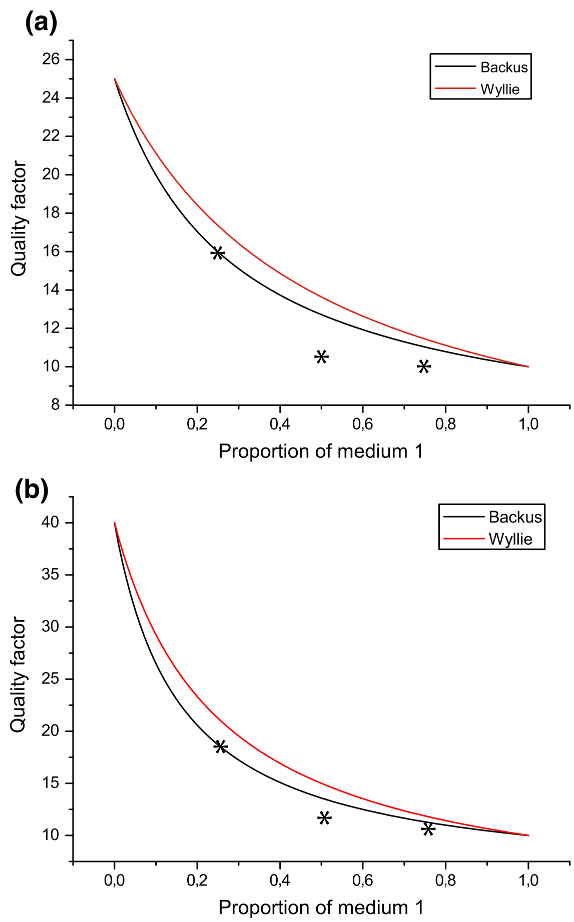

Figure 2
Backus and Wyllie quality factors as a function of p_1, where **a** and **b** correspond to Q-factor contrasts (Q_2/Q_1) of 2.5 and 4, respectively. The results obtained with Eq. (20) are shown as asterisks

shows Backus and Wyllie quality factors for Q_2 = 25 and Q_2 = 40, where it can be seen that the Wyllie average shows less attenuation. The asterisks are the results of the following simulations.

We perform 1D simulations to compare the results with Backus averaging and check the use of Eq. (9) to obtain averaged Q values at the long-wavelength limit. The number of grid points is 3465 and the grid size is 1 m. We consider ten sets of the previous media defined by the subindices 1 and 2 above (a distance 10 H) with different proportions p_1 = 0.25, 0.5, and 0.75, such that each set has four grid points (H = 4 m). The fine layers, therefore, are located between grid points 1732 and 1772. The source is a Ricker wavelet with a dominant frequency f_p equal to f_0 (e.g., Carcione 2014, p. 547) and is located at grid point 1731, while the receiver is placed at grid point 1773 (see Fig. 3). The algorithm stepping time is 0.1 ms. Figure 4 shows the wavefield for three different values of p_1 and Q_2 = 25. According to an exponential decay of the form $A = \exp[-10H\omega/(2c_B Q_B)]$, where 10 H is the source–receiver distance in this case, the quality factor can be approximately obtained as follows:

$$Q_B \approx -\frac{10H\pi f_0}{c_B \ln A}, \tag{20}$$

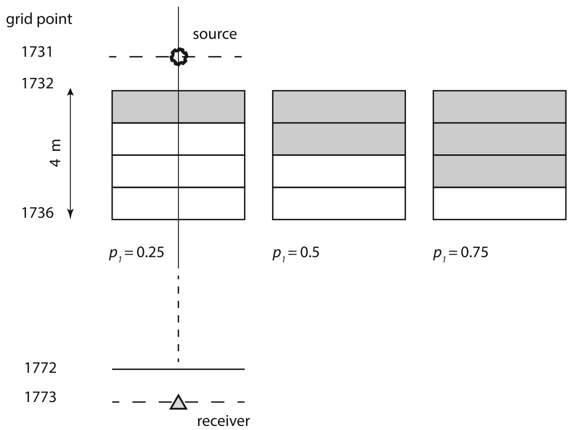

Figure 3
Discrete representations of the three modeled sets of fine layers, each of which is repeated ten times between grid points 1732 and 1772, with varying configurations of medium 1 (grey) and medium 2 (white)

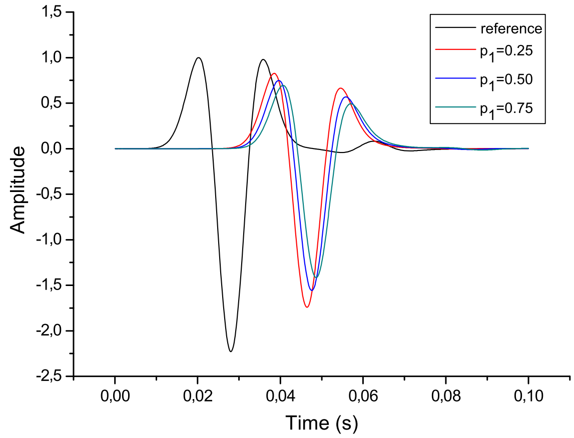

Figure 4

Wavefields at 40 m distance from the source for different values of p_1. The reference signal is the field at the source location

where A is the ratio between the maximum of the wavefield (at the receiver) and the maximum of the reference field (at the source). The results of the simulations are represented by asterisks in Fig. 2, showing that the agreement with the Backus curve is satisfactory, mainly for the case of higher contrast between Q factors. In summary, we have computed average Q values at two limits, i.e., Backus (low frequencies) and Wyllie (high frequencies).

It has to be clear that the asterisks in Fig. 2 are not the central result of the paper but an estimation of the Backus quality factor using an approximation. One could use the frequency-shift or the spectral-ratio methods, or any other method to obtain these values. We intended to show that the Backus average is a lower limit and this is reflected in Fig. 2. The central results of this work are the solid lines in this figure [obtained with Eqs. (9) and (17)]. In any case, even if equation (20) is an approximation, we can see that of the six points shown, three coincide with the curves and the other three are well below. This leaves no doubt that Eq. (9) provides, first, a lower limit and, second, agreement with Eq. (20). One problem could be the relatively small difference between the Backus and Wyllie averages, at least in the examples shown, which could be difficult to estimate in real data, mainly when the signal-to-noise ratio is low. However, this is a problem of the Q inversion algorithm.

6. *Conclusions*

Backus and Wyllie equations are expressions widely used to obtain average seismic velocities at the long and short wavelength limits, respectively. For instance, Backus averaging is used to downscale log data at seismic resolution and the Wyllie equation is often used for inferring porosity from well logs, as well as an in-situ indicator of pore fluid type. Here, those equations are generalized to obtain averages of seismic attenuation. The results indicate that the Wyllie phase velocity and quality factor are higher than the corresponding Backus quantities, where the first result (regarding the velocity) is that expected, since the ray velocity is a high-frequency limit. Since the quality factor can be related to properties such as porosity, permeability, and fluid viscosity (e.g., mesoscopic or wave-induced fluid-flow loss), these averages can be useful for evaluating reservoir properties.

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